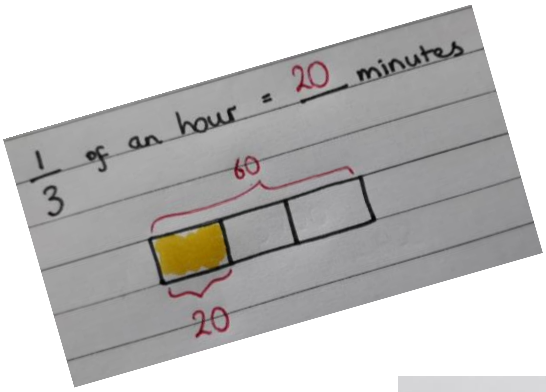
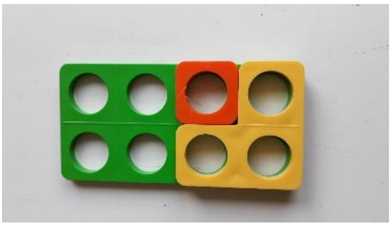
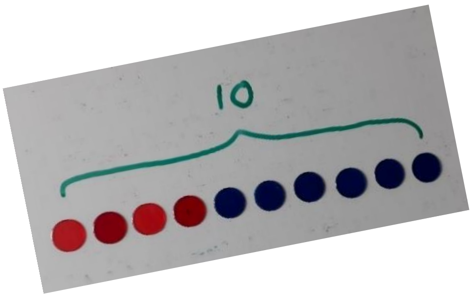


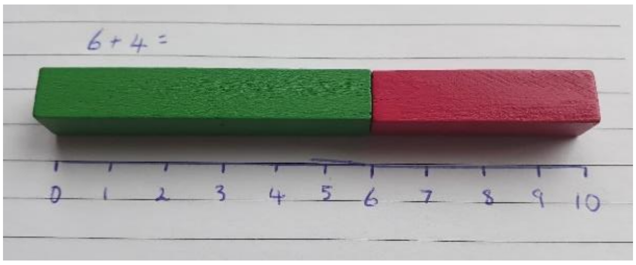


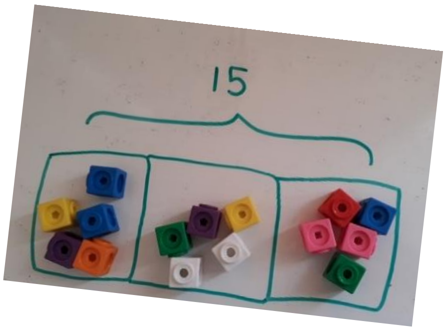
Bar Modelling

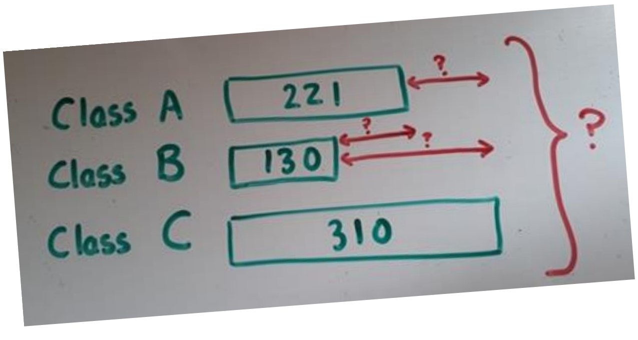
Whole School Progression Document

September 2020









Contents

What is bar modelling?

Bar modelling is designed to help children represent underlying structures and visualise maths problems. It was introduced in Singapore in the 1980s with an increased attention placed on problem solving.

In the 1960s Jerome Bruner proposed that people learn in three stages: *concrete, pictorial, abstract*. Bar models act as a bridge between concrete and abstract as they support children with the pictorial stage. In the concrete stage, the structure of a bar model can be explored using manipulatives. Using the pictorial bar model allows children to understand what they are being asked to do before then completing the calculation in the abstract form.

A bar model uses rectangles to represent known and unknown parts of a problem and places emphasis on understanding parts and wholes. They bring together all the parts of a question into one diagram. Once a student has represented all the necessary information and identified the unknown part, including which operation they may need to use, they can begin working out the solution (this is now the abstract stage). A bar model will not tell a child the answer to a problem but will help them understand the structure and what they are required to do.

Concrete

Pictorial

6	
4	2

Or

$$\overbrace{\boxed{\quad 4 \quad} \boxed{\ 2\ }}^{6}$$

Abstract

4 + 2 or 2 + 4

Different types of bar models:

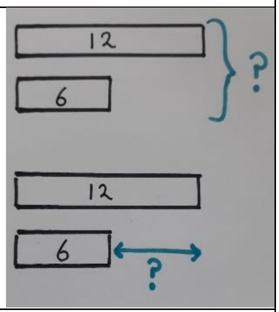
Part/whole bar models

whole	
part	part

The whole is the sum of the parts.

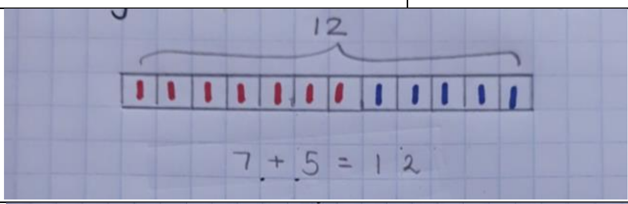
Comparison bar models

Comparing two amounts by drawing their bars and having brackets represent 'the whole' or 'the difference'.



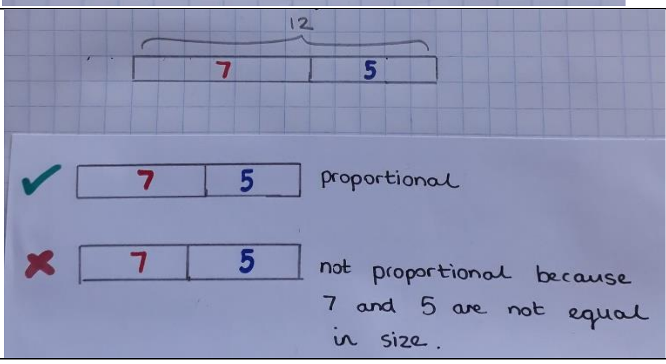
Discrete bar models

Every unit is an individual box.



Continuous bar models

Amounts are represented as proportional rectangles.

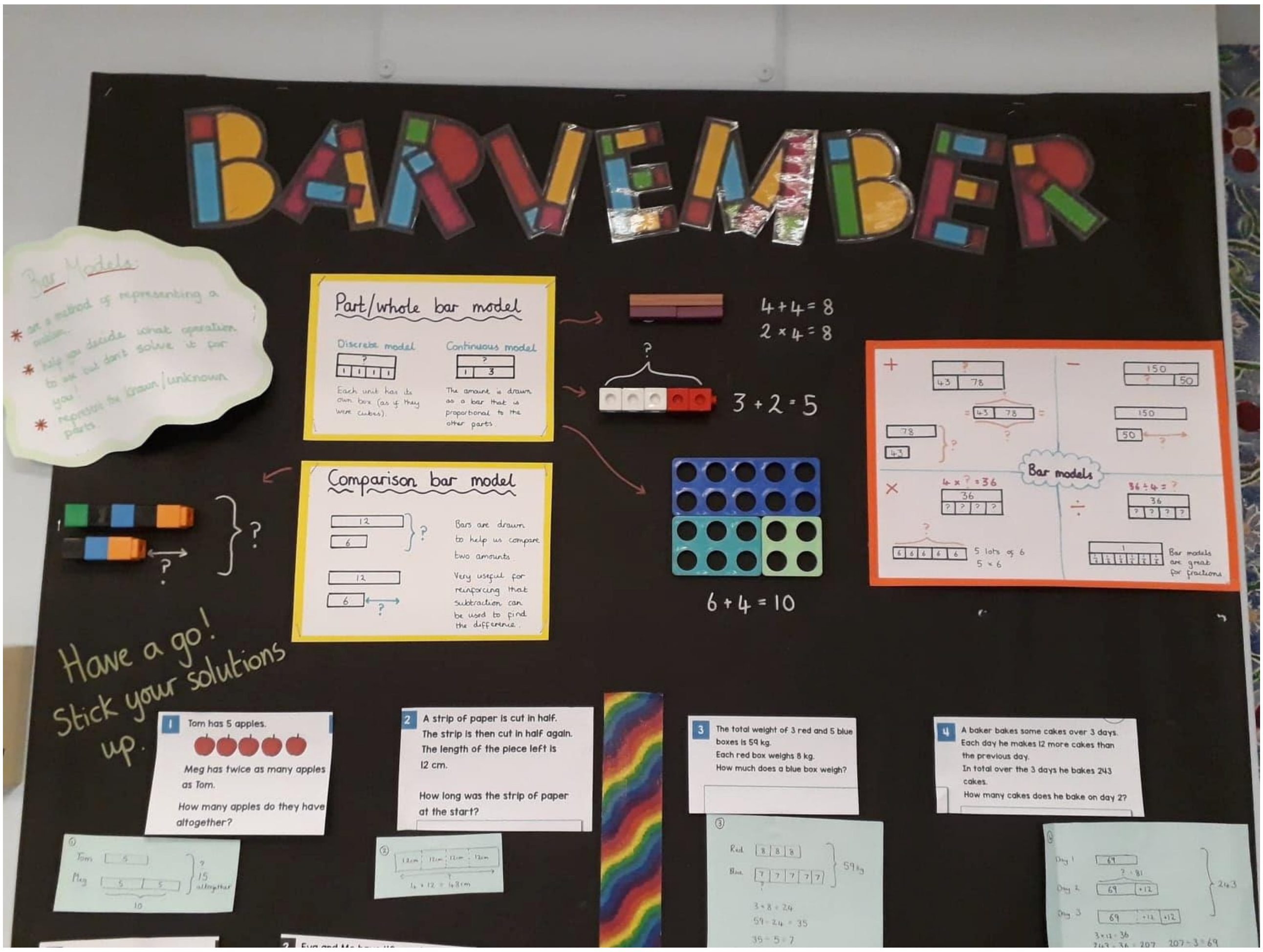


Many schools choose to adopt a bar modelling approach to problem solving to ensure children are equipped with a consistent, reliable and flexible tool for facing problems that are tricky to visualise. Bar models can be manipulated in both concreate and pictorial forms to help children establish what the known and unknown parts of their problem are. They can then use their bar model representation to decide what calculations will lead them to an answer.

BARVEMBER

https://whiterosemaths.com/resources/classroom-resources/barvember/

Every year, White Rose Maths hub host 'Barvember' which provides children with an opportunity to practice their skills and for children to explore different, creative approaches to bar modelling whilst also raising the profile of bar models. Staff are encouraged to join in with the challenges as part of their professional development. By being involved, staff will deepen their understanding of bar modelling which will ultimately enhance their ability and confidence to teach effectively using them.



Overview of teaching progression

Bar modelling structures and vocabulary are introduced to children in the Early Years Foundation Stage (EYFS). Throughout school, concrete representations of bar models should be used to support transition into pictorial representations.

In all year groups, the concrete manipulation of objects in linear structures to represent bars should be explored and understood sufficiently ***before*** introducing the pictorial representations that are shown in this document. Cubes, counters, objects and Cuisenaire rods are used to support exploration of bar model structures at the concrete stage of learning in ***all*** year groups when children come across new and more complicated structures. Similarly, even where children have used bar models before for that area of maths, teachers may choose to revisit the concrete stage to ensure a deep understanding of the structure before moving on.

Bar models can be adapted and varied in many ways but the underlying structures remain the same. Children need to see that they are a flexible tool by varying whether children are asked to 'find a part' or 'find the whole' when using bar model representation e.g.

<table>
<tr><th>Find a part</th><th>Find the whole</th></tr>
<tr>
<td>15 - ____ = 4
<table><tr><td colspan="2">15</td></tr><tr><td>4</td><td>?</td></tr></table></td>
<td>234 + 125 = ____
<table><tr><td colspan="2">?</td></tr><tr><td>234</td><td>125</td></tr></table></td>
</tr>
</table>

In EYFS and early year 1, use brackets above a bar to represent the whole. Towards the end of Year 1 and throughout Year 2, introduce using whole bars above the bar model to represent the whole; also continue to use the brackets so that the children do not forget that that is also an accurate representation. As children progress through KS2, they experiment with manipulating the bar model and representing the whole in different places (see addition section).

Progression in drawing of bar models:

EYFS	Year 1	Year 2	KS2
• Concrete exploration • Present items in a linear fashion. • Look at and discuss bar models with pictures in e.g. 5s and 10s frames • Not expected to draw accurate models independently though could start drawing boxes around objects like a bar model • Children should not be discouraged if they try to draw bar model jottings.	• Draw discrete bar models accurately and independently. • Use brackets for the whole but be exposed to diagrams where the whole is represented as a bar • Look at and discuss continuous models. • Begin to use continuous models where it becomes inefficient to draw discrete models.	Make a transition from discrete to continuous for most areas of maths and be able to draw these independently and accurately with increasing levels of proportionality.	Use continuous models with increasing levels of proportionality and variation in where the whole is depicted.

Progression in vocabulary of bar models:

EYFS	Year 1	Year 2	KS2
• Children should understand and identify parts and wholes. • Not expected to call them bar models.	• Children use part and whole vocabulary • Children can identify them as bar models	• Children confidently use part and whole vocabulary • Brackets terminology used when comparing whole bar to brackets drawn previously in year 1	Children can explain all aspects of a bar model, including parts/wholes, known/unknown and brackets/bars

By Y6, children should use everything that they have learned to help them understand the structures of any problem they are facing. They should be confident using the bar model to represent problems, identifying known and unknown parts and then choosing the appropriate method for calculating the answer.

Sometimes in this document the Year 6 column looks like they 'don't use' bar models. In fact, it is the complete opposite. Year 6 is the culmination of all of the exposure and work with bar models in earlier years; Year 6 is about confident and ***independent*** application of learned bar model structures, whatever the problem, and being able to manipulate the structures they have learned during their primary years.

Ensuring there is consistency in the teaching of specific vocabulary and representations of different bar model structures deepens children's understanding of bar models as a tool and enables them to be able to use them as an efficient tool for problem solving.

<u>Progression across the year groups</u>

<u>EYFS – bar modelling foundations</u>

For all of the following areas, progression begins with the use of real life objects and moves to cubes/counters. The final stage would be for children to draw boxes around objects to show they are parts of a bar.

Understanding number

In EYFS, the 5s frame (or 10s frame) can be used to stimulate mathematical talk and exposure to a 'bar' representing parts if the objects are placed in a linear fashion.

For example:

How many have we got? What is our whole? How many spaces are there? How many could we have?

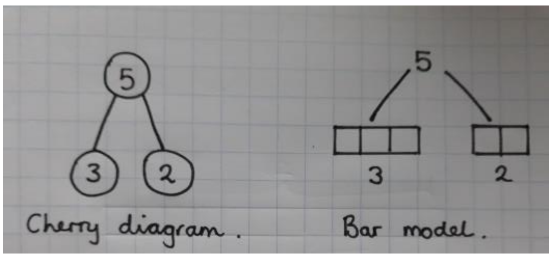
What do you notice? What's happened? Is this still 3? What is our whole?

What about now? Is our whole the same?

What has happened now? [there's another car]. How many have we got now? What is our number now? What is our whole? How many parts/spaces are left? Could we have any more? How many more could we have? Could we have two more?

What do you notice about this bar? This bar is full. How many have we got? What is our whole?

Representing number bonds



Using both of these representations for number bonds will ensure children are provided with variation in their representations and also begin to build foundations for independently drawing these in Year 1.

A large emphasis is placed on the part and whole vocabulary.

One more / less

- *Show me one more.*
- *Show me one less.*
- *How many do we have now?*
- *What is our whole?*
- *How many more can we have? Then how many would we have? What would our whole be?*

Add and subtract 2 single digit numbers

Using objects, children begin with a start number and then either add or take away a given number. Here, presenting the objects in a linear fashion allows for the early exposure of a 'bar' representation though it won't be referred to as that. Discussion will surround what the whole is and how many parts you added/took away.

 *3 add 2 equals 5. 5 is our whole. We added these two parts together.*

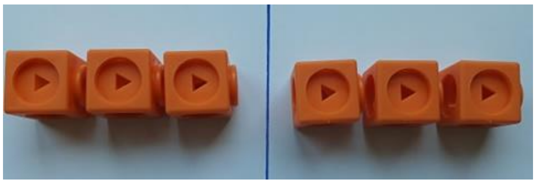
 *5 is our whole. 5 take away 1 is 4.* 

Doubling and halving.

Discussion surrounds the whole and the parts.

Doubling: 

We doubled this part [the four]. How many do we have now? 8 is our whole.

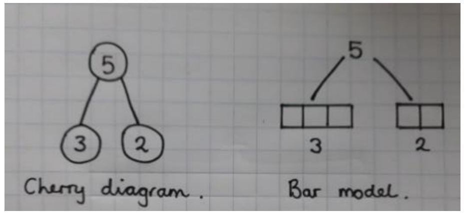
Halving: 

How many did we start with? 6 was our whole. We halved it [either splitting or sharing]. We have 2 parts now. Half of 6 is 3.

Place value

Year 1

Partitioning

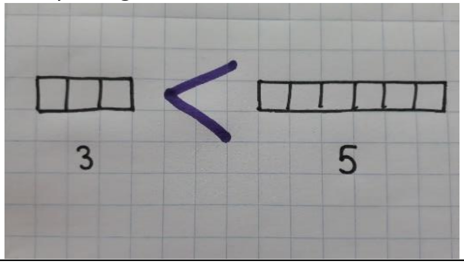


1	1	1	1	1

1	1	1	1	1

1	1	1	1	1

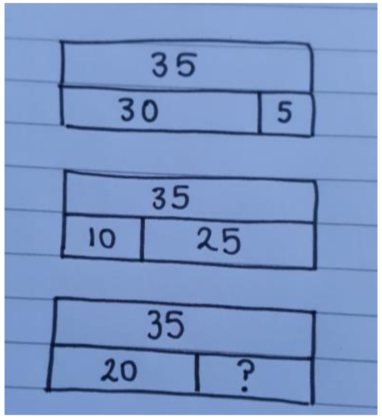
Comparing numbers



Year 2

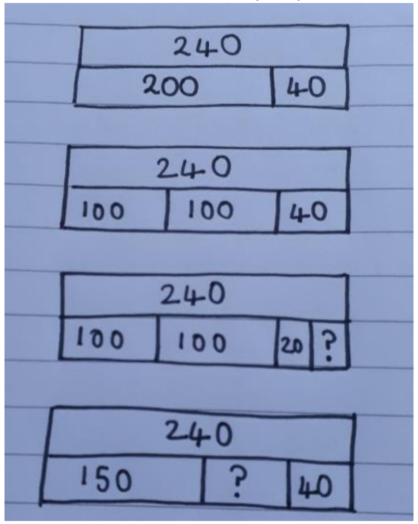
Use continuous models.

Partition numbers in different ways with the ‘unknown’ in different places.

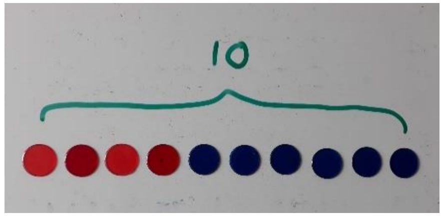


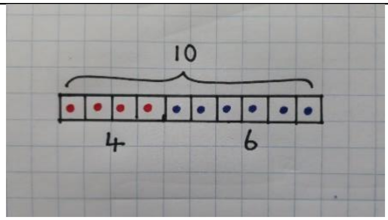
KS2

Partition numbers in different ways with the ‘unknown’ in different places.
Use increased levels of proportionality.



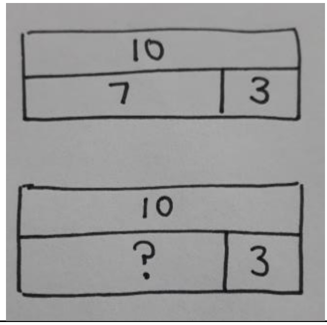
Number bonds

<table>
<tr><th>Year 1</th><th>Year 2</th><th>KS2</th></tr>
<tr>
<td>




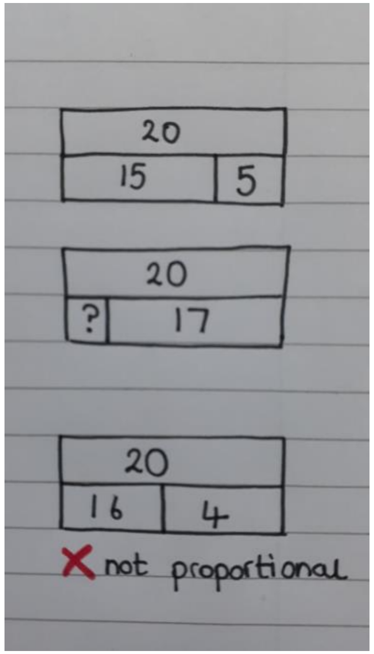
Place a huge emphasis on understanding what each part of the bar model shows: which are the parts, which is the whole?

Expose children, through effective teacher modelling, to continuous models when their number bonds are secure so that their working memory is not overloaded trying to work out the answer and interpret the new structure.



</td>
<td>
Use continuous bar models to develop fluency in number bonds to 20 and 100 and to show understanding of related subtraction facts by filling in the numbers on pre-drawn bar models.

Children should progress to be able to draw continuous bar models independently showing some degree in the understanding of proportionality.

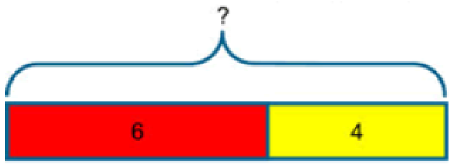


</td>
<td>KS2 continue to use bar model representations for number bonds when deemed appropriate.</td>
</tr>
</table>

Addition

There are 2 models for addition as shown. Where possible with the size of numbers, always begin with concrete representations and transition to the pictorial bar model when this becomes inefficient with concrete materials.

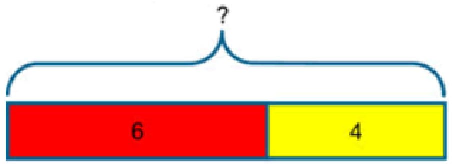
Addition
Aggregation
- two quantities combined



I have 6 red pencils and 4 yellow pencils. How many pencils do I have?

(I combine two quantities to form the whole)

Addition
Augmentation
- a quantity is increased



I have 6 red pencils and I buy 4 yellow pencils. How many pencils do I have?

(The bar I started with increases in length)

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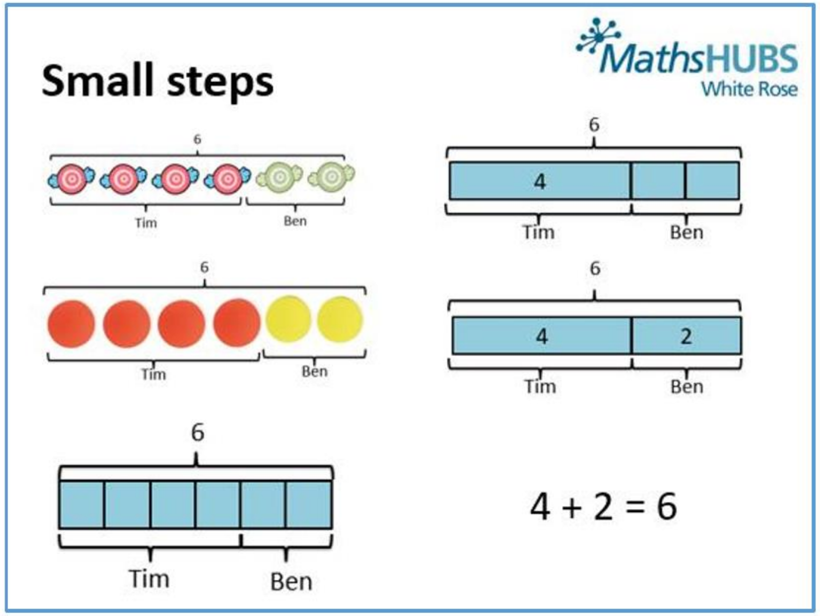
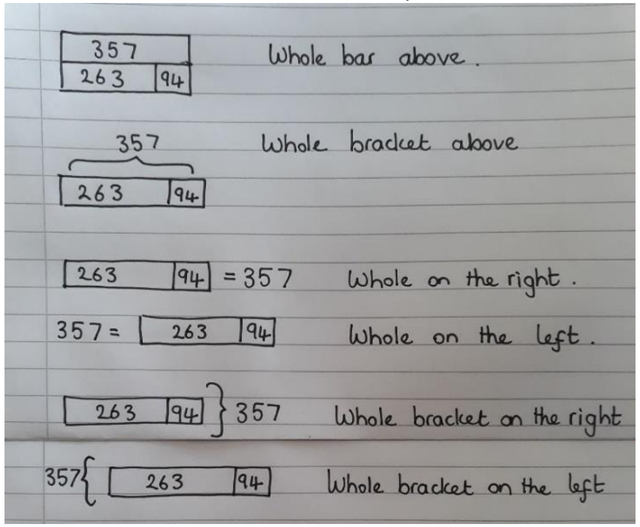
<table>
<tr><th>Year 1</th><th>Year 2</th><th>KS2</th></tr>
<tr>
<td>
</td>
<td>Use the vocabulary:
4 is a part.
2 is a part.
The whole is 6.

By Year 2, when dealing with 2 digit numbers, continuous models should be used to ensure efficient calculation.

Cubes / counters / Cuisenaire used to transition from real objects to pictorial bars.</td>
<td>Variation in numbers and representations


'Whole below' is less conventional though children should understand that it is not incorrect.</td>
</tr>
</table>

Use this progression for:

- *Adding numbers within 10*
- *Fact families*

__ + __ = 7　　7 = __ + __

__ + __ = 7　　7 = __ + __

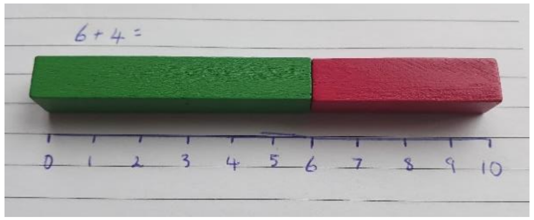
[White Rose Y1 planning document]

- *Adding groups together (aggregation)*
- *Adding more (augmentation)*
- *Adding two numbers within twenty*

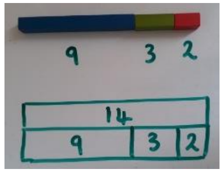
16 + 2 =

16		

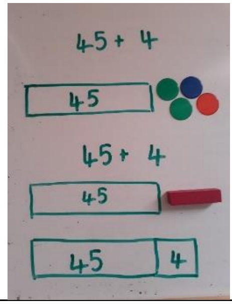
Number bonds and adding numbers (particularly adding on) could also be shown on number lines with bars above (using Cuisenaire) **if** the children are confident in their understanding of both number lines and parts/wholes.



- *Adding 3 one digit numbers (could be done as augmentation or aggregation)*



- *2 digit numbers and ones (could be done as augmentation or aggregation)*



Use the continuous bar model consistently for representing:

- *2 digit number and tens*

Use continuous bars, with increasing proportionality.
e.g. 45 + 10

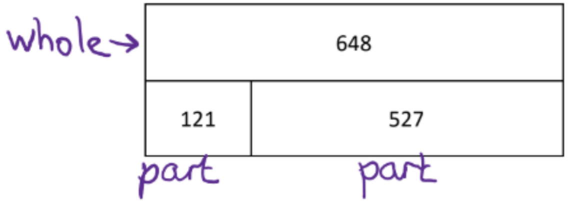
55	
45	10

- *2 two digit numbers*

e.g. 45 + 24

59	
45	24

Use bar models to understand inverse relationships.

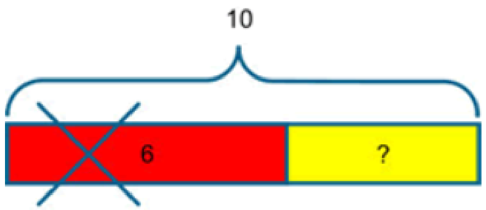


527 + 121 = 648
121 + 527 = 648
648 – 121 = 527
648 – 527 = 121

527 – 121 = 648

This would NOT be a correct sentence because 527 – 121 would equal 406.

Subtraction – take away



I had 10 pencils and I gave 6 away, how many do I have now?

(This time we know the whole but only one of the parts, so the whole is partitioned and one of the parts removed to identify the missing part)

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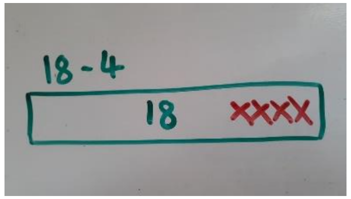
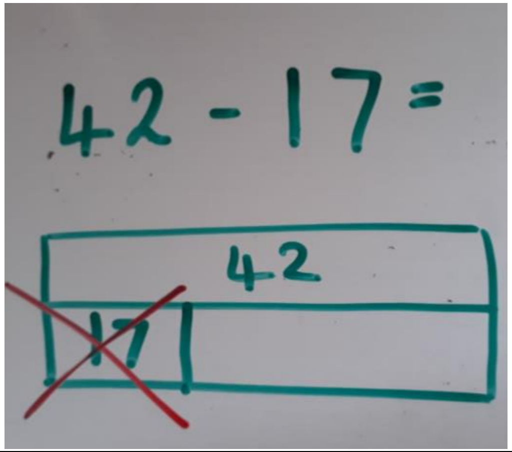
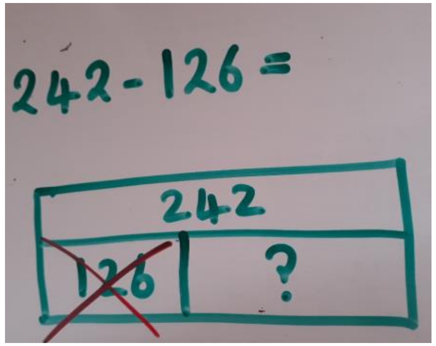
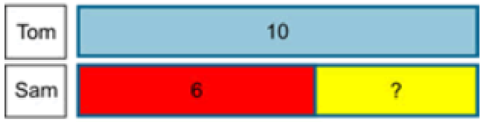
<table>
<tr><th>Year 1</th><th>Year 2</th><th>Year 3</th><th>Year 4</th><th>Year 5</th><th>Year 6</th></tr>
<tr>
<td>Discrete model
7 -4
| | | | x | x | x | x |

Draw on a continuous model and count back

</td>
<td>Continuous model

</td>
<td colspan="4">Continuous model with variation of number.

</td>
</tr>
</table>

Subtraction – finding the difference

Subtraction
- Comparison or Difference



Tom has 10 pencils and Sam has 6 pencils. How many more does Tom have?

(The bar is particularly valuable for seeing the difference between the two quantities)

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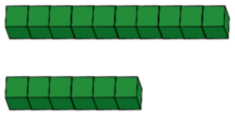
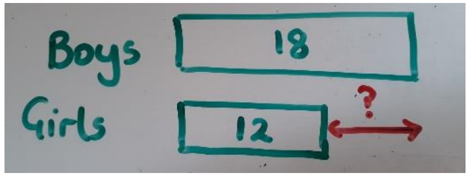
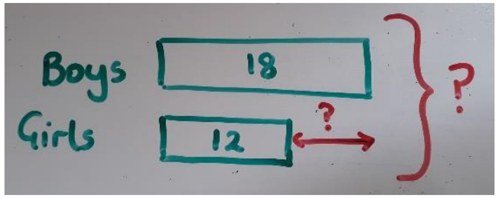
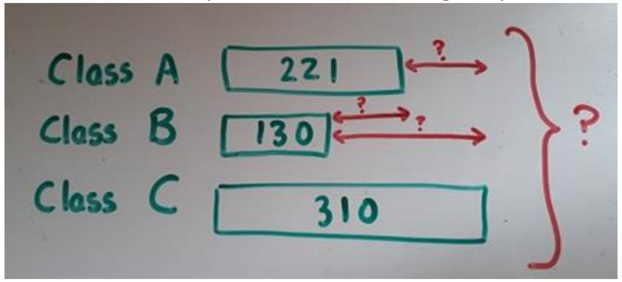
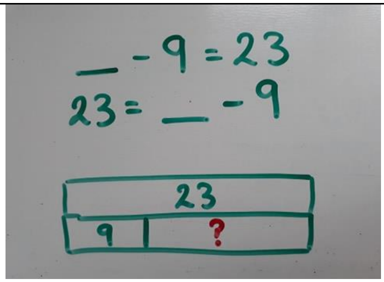
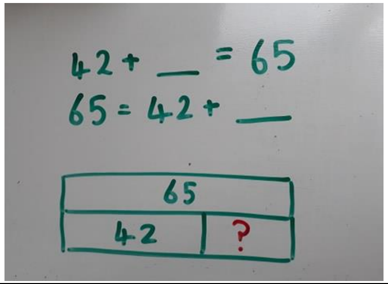
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Use concrete apparatus in linear fashion to compare the sizes. Identify the gap representing the difference. Discuss how many more / how many less.  What's the difference between 10 and 6? The difference between 10 and 6 is ___ 10 – 6 = ___ [Y1 White Rose document] Children need to be confident with the vocabulary surrounding finding the difference as subtraction.	Use comparison continuous models to find the difference and also to find the whole.   How many more boys are there in the class than girls? Discuss all the information we know: • There are 18 boys, 12 girls • There are 30 in total • There are 6 more boys • There are 6 fewer girls	Use comparison continuous models to find the difference, find the whole with numbers ≥ 3 digits. Also compare more than 2 groups.  			

Addition and subtraction – missing number problems

Once children are using the bar model with the whole as a bar at the top in Year 2, they can begin using bar models to represent missing number problems providing they have a secure understanding of how to interpret the parts, the whole and the unknown part of the question.

<table>
<tr><th>Year 2</th><th>Year 3</th><th>Year 4</th><th>Year 5</th><th>Year 6</th></tr>
<tr>
<td>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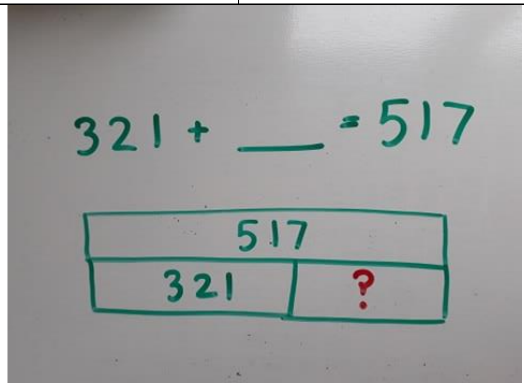
<td colspan="2">
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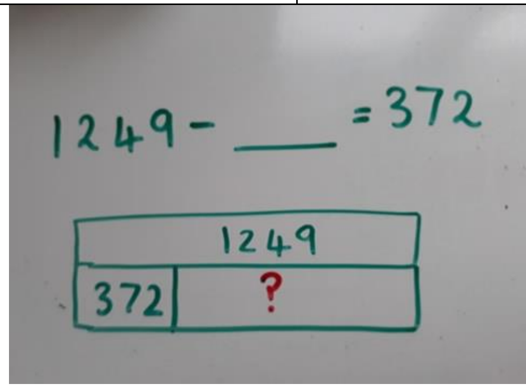
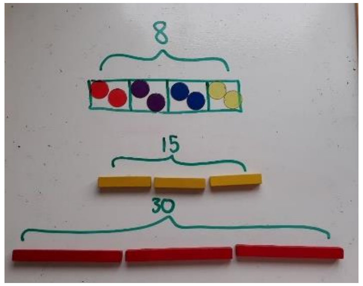
<td colspan="2">
</td>
</tr>
</table>

Multiplication

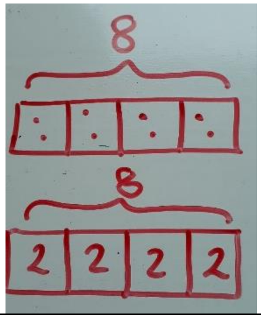
A large emphasis is placed on equal sized parts and children understanding multiplication as repeated addition.

<table>
<tr><th>Year 1</th><th>Year 2</th><th>Lower KS2</th><th>Upper KS2</th></tr>
<tr>
<td>

- *Count in multiples of 2s, 5s, 10s.*



Discuss repeated addition. Continuous models work well here as counting groups of 2s, you make sure one group goes in one box.



</td>
<td>

- *Count in multiples of 2, 3, 5, 10*

Follow Year 1 sequencing using Cuisenaire rods/counters/cubes and progressing to use continuous models using bars for the top whole.

Draw the parts first as you count up in the number:

<table><tr><td>5</td><td>5</td><td>5</td><td>5</td><td>5</td></tr></table>

Then add the whole bar on top:

<table><tr><td colspan="5">25</td></tr><tr><td>5</td><td>5</td><td>5</td><td>5</td><td>5</td></tr></table>

</td>
<td>

As with Year 1 and 2 but with different numbers.

- *Y3 > count in multiples of 4, 8, 50 and 100*
- *Y4 > count in multiples of 6, 7, 9, 25 and 1000*

</td>
<td>

Use the structure of repeated addition bar models to help understand and represent questions but use formal written methods to calculate answers.

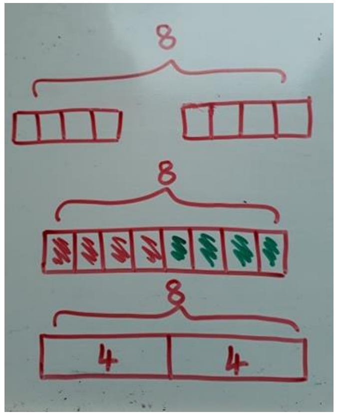
For calculations such as 43 x 28, a bar model would not be suitable. This is an arithmetic question and best suited for short multiplication.

Bar models could be used to *represent* problems such as:
Irvin bought 6 bags of apples, each weighing 132kg.

<table><tr><td colspan="6">?</td></tr><tr><td>132</td><td>132</td><td>132</td><td>132</td><td>132</td><td>132</td></tr></table>

</td>
</tr>
</table>

- *Doubling*



- *recall and use multiplication facts for the 2, 5 and 10 time tables*

Begin using 'groups of'
e.g. 3 x 5 is 3 groups of 5

15		
5	5	5

When children have learned that multiplication is commutative, they can become confident representing the number statement both ways

e.g. 4 x 10 is 4 lots of 10

40			
10	10	10	10

4 x 10 is 10 lots of 4

40									
4	4	4	4	4	4	4	4	4	4

- *recall and use multiplication facts for the 3, 4 and 8 time tables (Y4 – up to 12 x 12)*

Represent calculations in different ways depending on the word of a worded question.

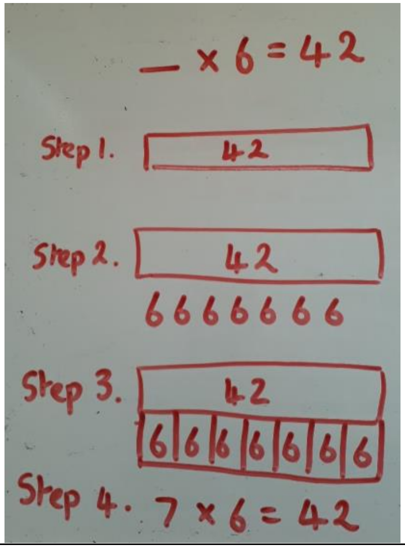
3 x 8 could be:
3 lots of 8 (8, 3 times)

24		
8	8	8

Or
8 lots of 3 (3, 8 times)

24							
3	3	3	3	3	3	3	3

- *Solving missing number problems using multiplication knowledge*

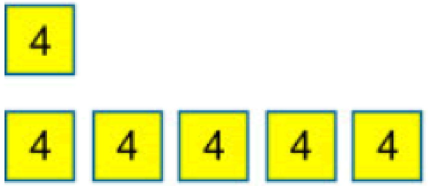


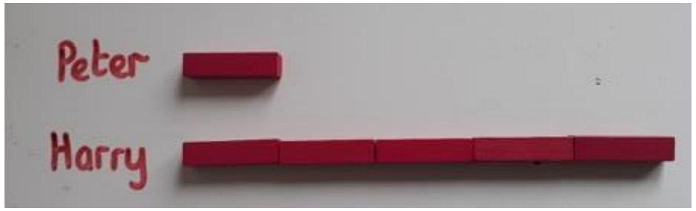
_____ x 6 = 42
6 x ___ = 42
42 = ___ x 6
42 = 6 x _____

We know the whole is 42.
We are counting up in 6s.
Write out your 6s until you reach 42.
How many 6s did you write?
7 lots of 6 equals 42.

- *Solve problems using multiplicative relationships (linked to scaling integers)*

Peter has 4 books
Harry has five times as many books as Peter.
How many books has Harry?





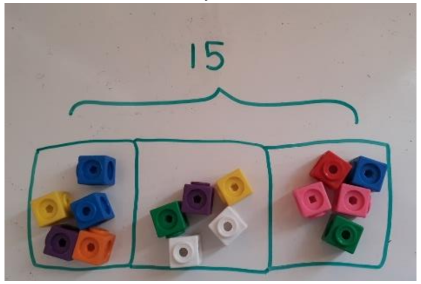
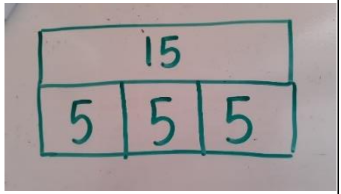
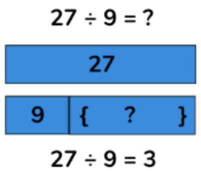
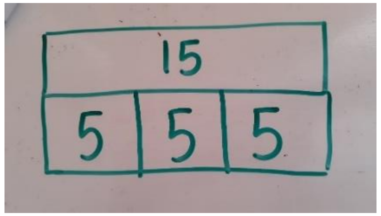
This could be introduced first using counters/cubes/Cuisenaire rods.
Further questioning:

- How many more does Harry have than Peter? How many fewer does Peter have than Harry?
- How many do they have in altogether?

Division

Bar model representations of division are dependent on the wording used in the question.

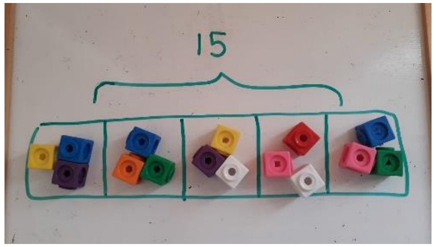
If it uses the division symbol default to 'sharing' - particularly in Years 1 and 2. As children become familiar with both the grouping and sharing bar models, they may develop a preference for solving calculations that use the division symbol but should know that if it is a worded problem, they will need to select sharing or grouping accordingly.

Year 1	Year 2	Lower KS2	Upper KS2
• *Grouping* 15 cubes into groups of 5. How many groups? Use the manipulatives and then put boxes around them to create a bar. Demonstrate the whole at the top.  	• *Grouping* 15 into groups of 5. How many groups? Similar to counting in multiples and should use concrete manipulatives first. Draw the whole bar as 15. Count up in 5s. Stop when you get to 15. How many groups are there?   [Third space learning]	• *Grouping* 15 into groups of 5. Use times tables knowledge. 15÷5 = 3. I need 3 parts, each with 5 in them.  When introducing new times tables, use manipulatives first.	As with multiplication, bar models can be used to help represent and understand the structures of a question but would not be suitable for arithmetic questions such as 324 ÷ 6 if the child is going to 'count up' in 6s as this would be inefficient. Here, we would encourage them to use written methods of division. However, bar models could still be used to show an understanding of worded problems e.g. There are 324 chairs to put in the hall. The headteacher wants to put them in 6 rows. How many chairs will be in each row? Children could represent it as: 324 ? ? ? ? ? ?

<table>
<tr>
<td>

- *Sharing*

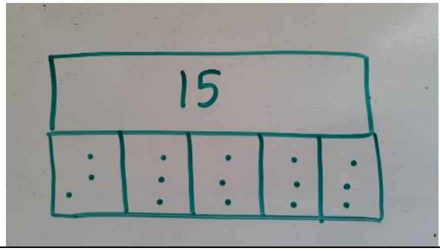
15 cubes shared between 5 friends.
How many does each person get?
Use the cubes and then draw the boxes to create the bar and mark the whole at the top.



</td>
<td>

- *Sharing*

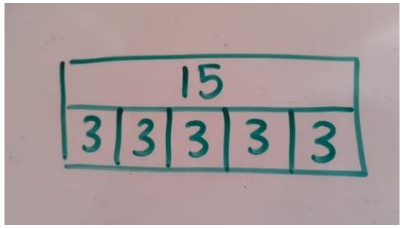
15 cubes shared between 5 friends.
Show 15 as the whole bar.
Split the bottom bar into 5, 1 part for each friend.
Count out the 15 across each part – remember division must be equal parts.



</td>
<td>

- *Sharing*

15 shared between 5 friends.
Use times tables knowledge.
15÷5 = 3.
Each person will get 3.
I need 5 parts, each with 3 in them.



When introducing new times tables, use manipulatives first.

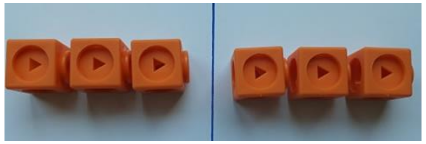
</td>
<td rowspan="2">

And then use written methods to find the size of the parts.

</td>
</tr>
<tr>
<td>

- *Halving*

Reinforce EYFS work



How many did we start with? 6 was our whole. We halved it [either splitting or sharing]. We have 2 parts now. Half of 6 is 3.

Progress to the children drawing two boxes and being able to share the counters out, or share by putting dots in the boxes.

</td>
<td colspan="2">

- *Division symbol*

e.g.
20 ÷ 5 =

Children can choose their preferred method but if unsure, the sharing method should be favoured until their counting in multiples is secure enough to support grouping.

If the number becomes large, choosing the most efficient method is important. You do not want children counting out 50 dots in order to divide by 5. It would be more efficient to use the grouping method and count up in multiples of 5.

</td>
</tr>
</table>

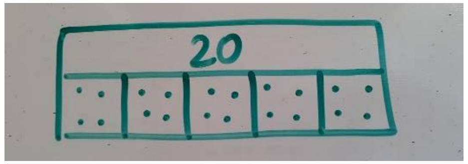
<table>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td>

• *Solving missing number problems: part unknown*

20 ÷ ___ = 5
Children will be taught to solve this by counting up in 5s, thinking about how many groups of 5 it is.

<table>
<tr><td colspan="4">20</td></tr>
<tr><td>5</td><td>5</td><td>5</td><td>5</td></tr>
</table>

If a child chooses to represent it as 5 parts and then share the counting out, this is not incorrect.



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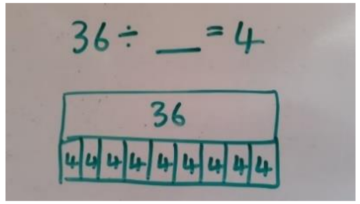
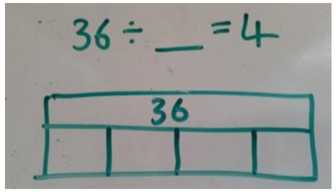
• *Solving missing number problems: part unknown*

e.g. 36 ÷ _____ = 4
We know the whole is 36



We can either say:
‘we know there are 4 in each group/part, so how many groups/parts’

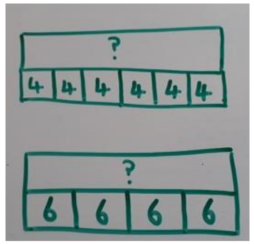
Or
‘we know there are 4 groups/parts in total, so how many in each group/part’?



</td></tr>
<tr><td></td><td></td><td>

• *Solving missing number problems: whole unknown*

______ ÷ 6 = 4



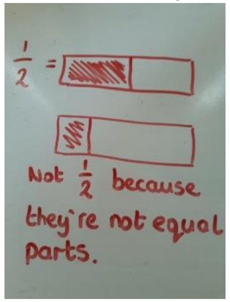
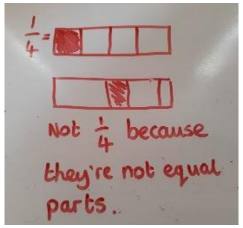
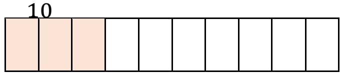
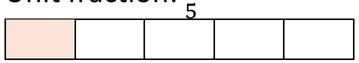
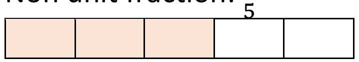
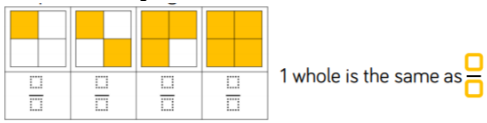
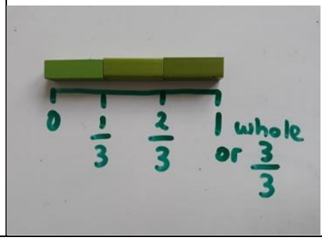
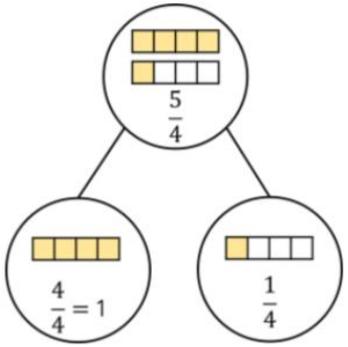
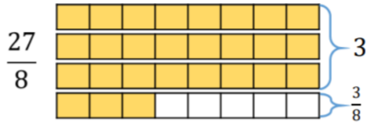
There are 6 groups, each with 4 in:

Or
Each parts has 6 in it and there are 4 parts.

Make strong links here to multiplication as repeated addition and use of times tables to find the whole, highlighting the inverse relationship between x and ÷

</td></tr>
</table>

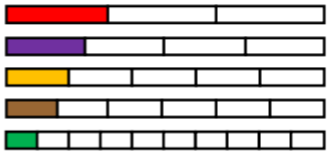
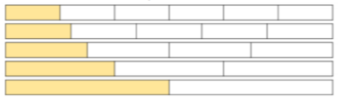
Fractions – representing fractions

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
• *recognise, find and name a half*  • *recognise, find and name a quarter*  Emphasis is placed on understanding fractions are parts of a whole.	• *recognise, find, name and write fractions* $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ *including unit and non-unit.*  $\frac{3}{4}$ The whole has been split/divided into 4 parts and we are looking at 3 parts OR there are 3 shaded.	• *count up and down in tenths* • *recognise that tenths arise from dividing an object into 10 equal parts* $\frac{3}{10}$  • *recognise and use fractions as numbers: unit and non-unit fractions with small denominators* Unit fraction: $\frac{1}{5}$  Non unit fraction: $\frac{3}{5}$  • *making the whole*  [White Rose Y3 planning document] • *fractions on a numberline* 	• *Fractions greater than 1 whole*  There are ___ quarters altogether. ___ quarters = ___ whole and ___ quarter. [White Rose Y4 planning document.]	• *convert from improper fractions to mixed numbers*  [White Rose planning document]	

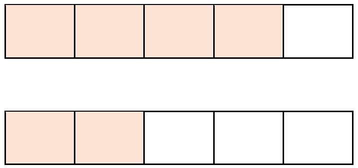
Fractions – comparing fractions

<table>
<tr><th>Year 2</th><th>Year 3</th><th>Year 4</th><th>Year 5</th><th>Year 6</th></tr>
<tr>
<td>Teacher might discuss that halves are bigger than thirds and quarters by showing bar models. Though this should be done with caution so that children do not think ½ is always bigger; it is dependent on the size of the whole.</td>
<td colspan="2">Use fraction walls where possible and Cuisenaire rods to support understanding.

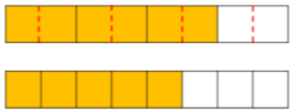
- *compare and order unit fractions (same numerator)*

Use >, < or = to compare the fractions.

$\frac{1}{10} \bigcirc \frac{1}{4}$ $\frac{1}{3} \bigcirc \frac{1}{6}$ $\frac{1}{5} \bigcirc \frac{1}{4}$
Now order the strips from the smallest to the largest fraction.

When the numerators are the same, the ______ the denominator, the _____ the fraction.
[Y3 White Rose document]

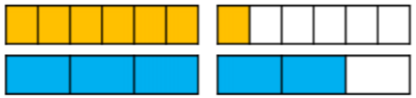
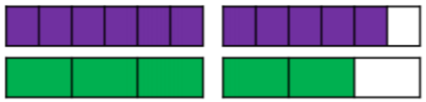
- *compare and order fractions with the same denominators*


$\frac{4}{5} > \frac{2}{5}$</td>
<td>

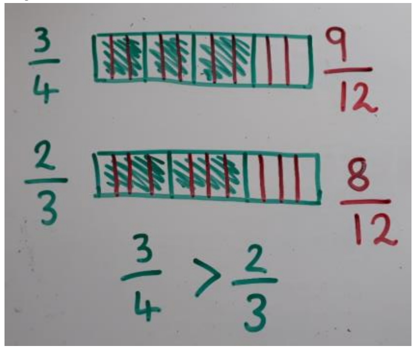
- *compare and order fractions less than 1*

Use bar models to compare $\frac{5}{8}$ and $\frac{3}{4}$

[Y5 White Rose document]

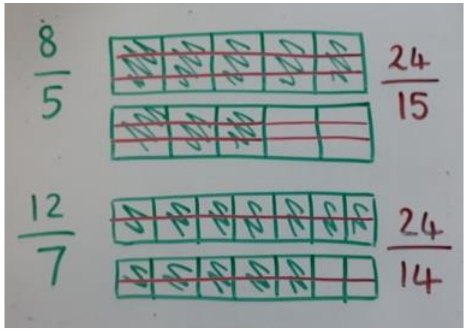
- *compare and order fractions greater than 1*

Use bar models to compare $\frac{7}{6}$ and $\frac{5}{3}$

Use a bar model to compare $1\frac{2}{3}$ and $1\frac{5}{6}$

[Y5 White Rose document]</td>
<td>

- *compare and order (denominators are not multiples of the same number)*



- *compare and order (numerator)*



These methods will be used to introduce and embed the structure of comparing fractions with different denominators and then children will move to the abstract form of finding common numerators and common denominators by using multiplication.</td>
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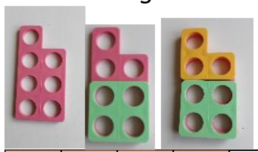
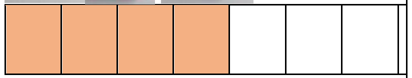
</table>

Fractions – adding fractions

Year 3

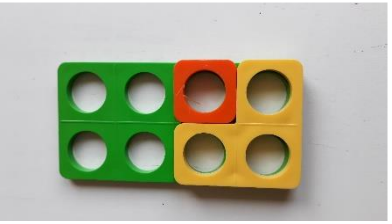
- *making the whole*





$\frac{4}{7}$ and $\frac{3}{7}$ make the whole $\frac{7}{7}$

- *adding fractions*



We can use this model to calculate $\frac{3}{8}+\frac{1}{8}=\frac{4}{8}$

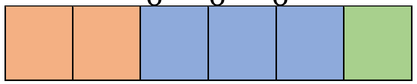
[Y3 White Rose document]

Year 4

- *add two or more fractions*
-

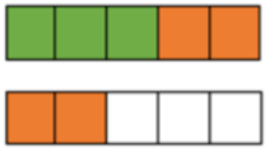
$$\frac{2}{8}+\frac{3}{8}+\frac{1}{8}$$

$$\frac{2}{6}+\frac{3}{6}+\frac{1}{6}$$



- *adding fractions and recording the answer using an improper fraction when the answer is greater than 1 whole*

$$\frac{3}{5}+\frac{4}{5}=\frac{7}{5}$$



[Y4 White Rose document]

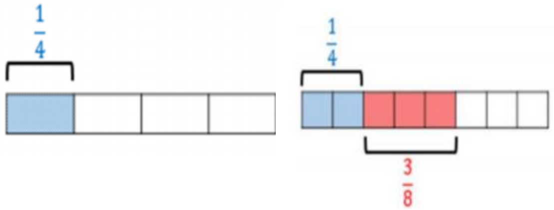
Year 5

$\frac{3}{5}+\frac{4}{5}=\frac{7}{5}=1\frac{2}{5}$

- *add fractions within one*

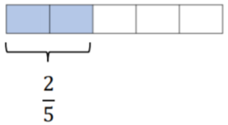
$$\frac{1}{2}+\frac{1}{8}=\frac{4}{8}+\frac{1}{8}=\frac{5}{8}$$

$$\frac{1}{4}+\frac{3}{8}=\frac{2}{8}+\frac{3}{8}=\frac{5}{8}$$



- *add 3 or more fractions*

$$\frac{2}{5}+\frac{1}{10}+\frac{3}{20}$$

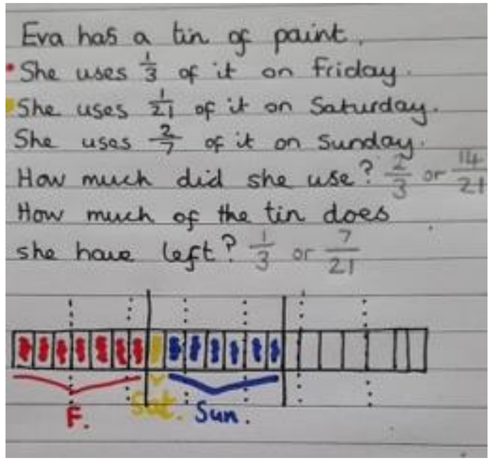


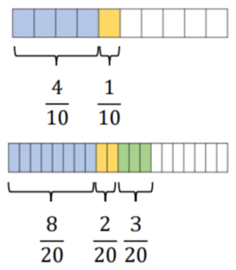
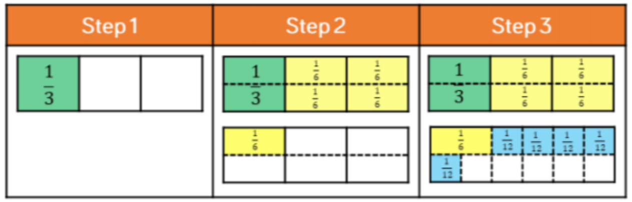
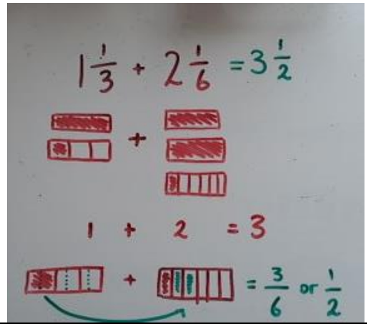
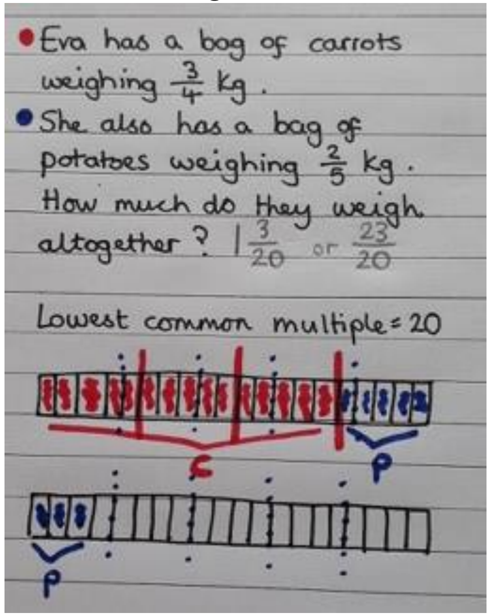
Year 6

Building on learning from Year 5, children learn to add and subtract fractions within 1 where the children need to find the lowest common multiple in order to find a common denominator (this could be practiced through bar model work as seen in Year 5).

Use the bar model to represent increasingly complex problems where common denominators need to be found.

Answers within one:



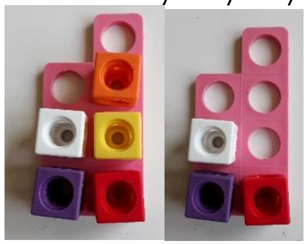
		 • *add 3 fractions where the answer is greater than 1*  $\frac{1}{3}+\frac{5}{6}+\frac{5}{12}=1\frac{7}{12}$ [all above images from Y5 White Rose document] • *add mixed numbers* 	Answers greater than 1: 

Fractions – subtracting fractions

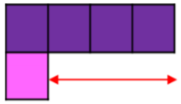
Year 3

- *subtract fractions with the same denominator within 1 whole*

$$\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$$



$$\frac{5}{7} - \frac{\square}{7} = \frac{\square}{7}$$



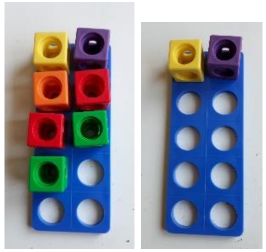
$$\frac{4}{8} - \frac{\square}{8} = \frac{\square}{8}$$

[Y3 White Rose document]

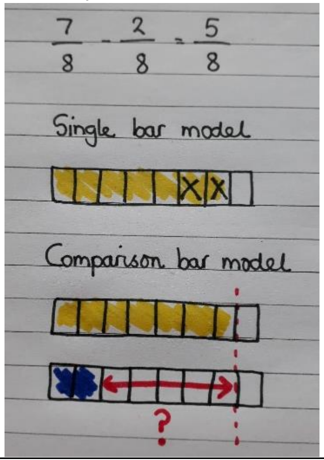
Year 4

- *subtract fractions with the same denominator*

$$\frac{7}{10} - \frac{5}{10} = \frac{2}{10}$$



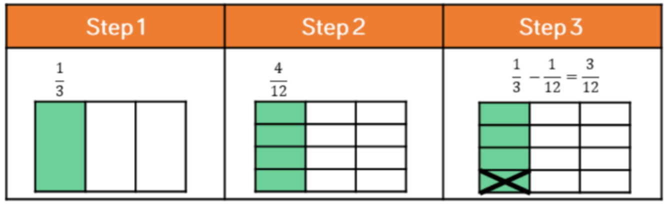
Children should be confident representing the subtraction as both a single bar model and a comparison bar model.



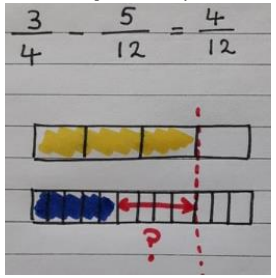
Year 5

- *Subtract fractions with different denominators*

Using a single bar model:

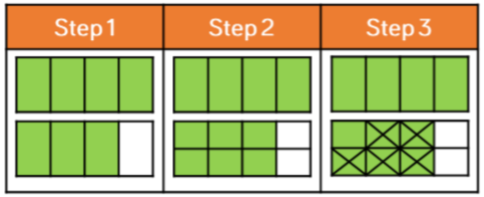


Or using a comparison model:



- *Subtract mixed numbers*

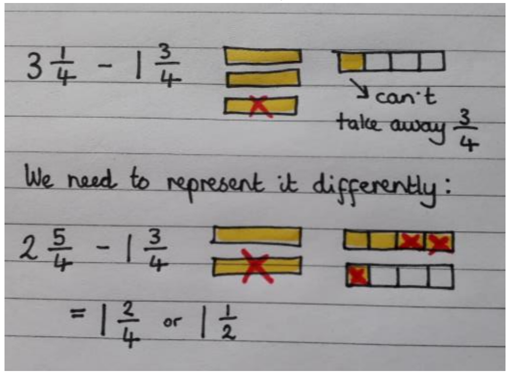
$$1\frac{3}{4} - \frac{5}{8} = 1\frac{1}{8}$$



$$2\frac{3}{4} - \frac{7}{8}$$

Year 6

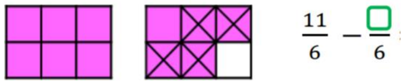
Continue to embed exchange a whole bar for a bar of fractions as shown in the final Year 5 example and below:



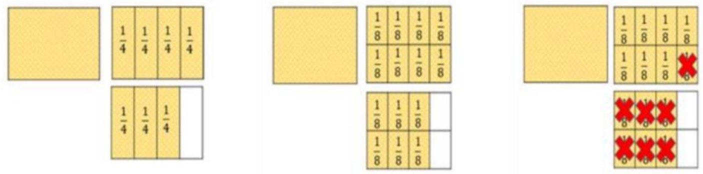
Apply bar modelling representations to help tackle scenario problems.

On Monday she eats $\frac{2}{3}$ of a bag and gives $\frac{4}{5}$ of a bag to her friend.
On Tuesday she eats $1\frac{1}{3}$ bags and gives $\frac{2}{5}$ of a bag to her friend.
What fraction of her sweets does Alex have left?

- *(including subtracting from fractions greater than 1 whole)*

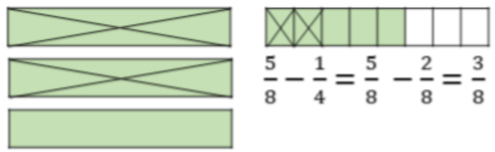


[Y4 White Rose document]



- *Subtract 2 mixed numbers*

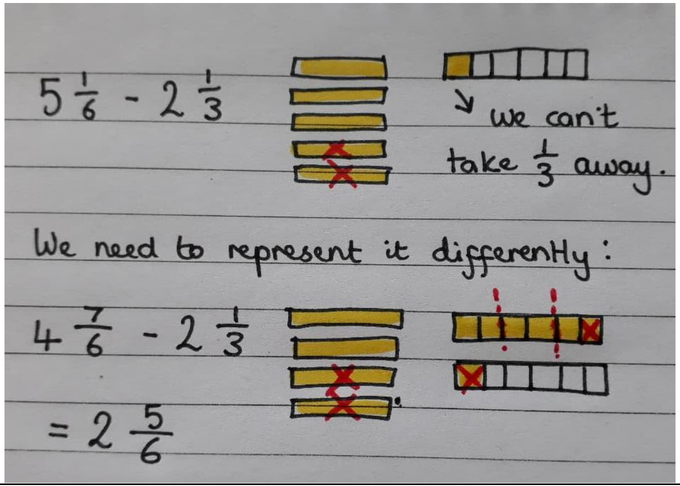
$$3\frac{5}{8} - 2\frac{1}{4}$$

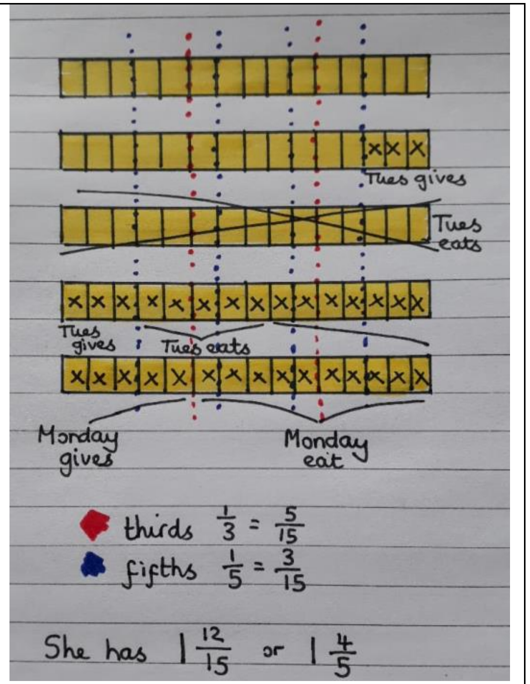


$$\frac{5}{8} - \frac{1}{4} = \frac{5}{8} - \frac{2}{8} = \frac{3}{8}$$

$$3 - 2 = 1$$

[all above images from Y5 White Rose document]





Fractions – equivalent fractions

Year 2

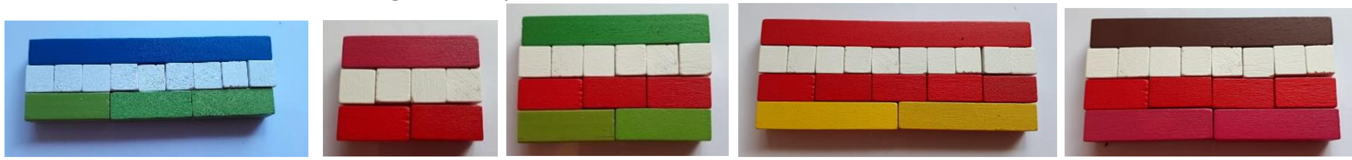
- Recognise the equivalence of $\frac{1}{2}$ and $\frac{2}{4}$

Additional manipulatives could be used, such as fraction circles.

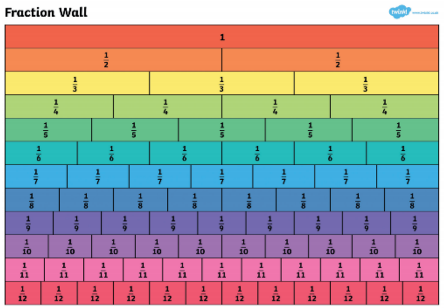
Year 3 / Year 4

- Y3 - recognise and show, using diagrams, equivalent fractions with small denominators
- Y4 - recognise and show, using diagrams, families of common equivalent fractions

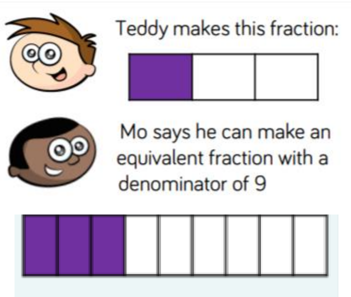
Use Cuisenaire rods to investigation equivalent fractions.



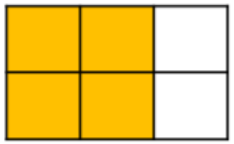
Progress to being able to identify equivalent fractions from a pictorial fraction wall where the rows can be treated as 'bars'.



Use the bar model to apply understanding of equivalent fractions through the use of diagrams:



Explain how the diagram shows both $\frac{2}{3}$ and $\frac{4}{6}$



[Y3 White Rose document]

Year 4 > use the bar model as a precursor for recognising the multiplicative relationship between equivalent fractions.

Using the diagram, complete the equivalent fractions.

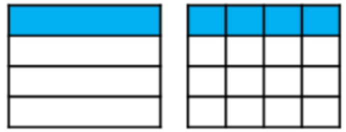
$\frac{1}{4} = \frac{\square}{12}$ $\frac{1}{\square} = \frac{6}{12}$ $\frac{2}{3} = \frac{\square}{12}$ $\frac{5}{12} = \frac{\square}{24}$

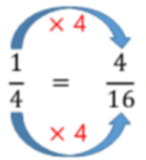
[Y4 White Rose document]

Year 5 / Year 6

Revisit concrete and pictorial exploration.

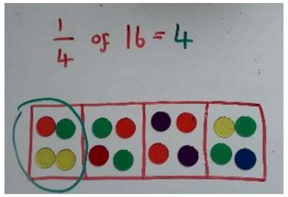
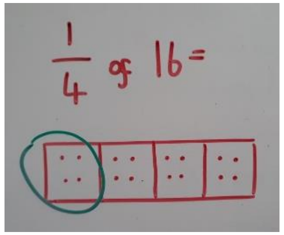
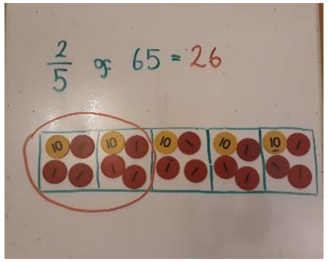
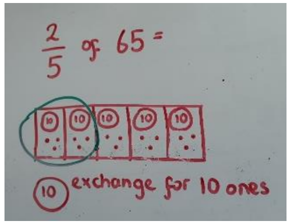
Use models to represent equivalent fractions and illustrate their multiplicative relationship.

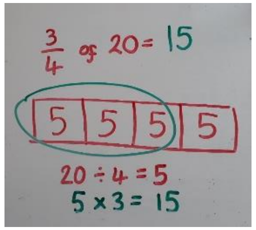


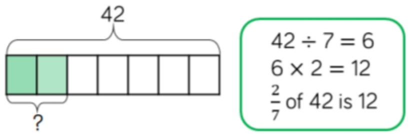
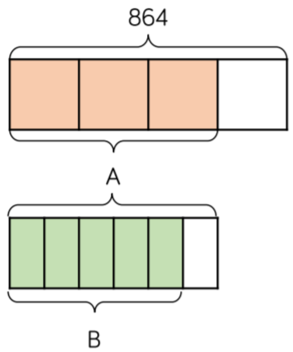
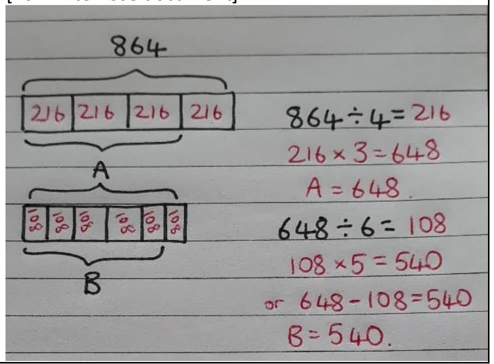


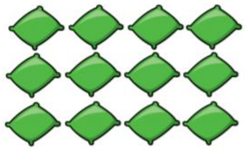
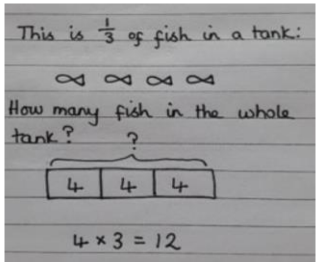
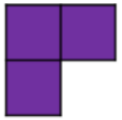
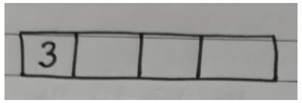
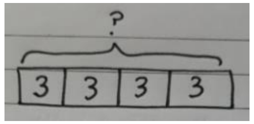
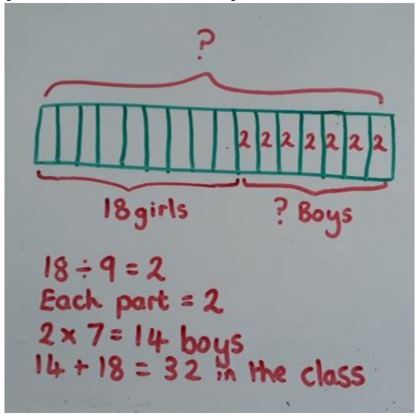
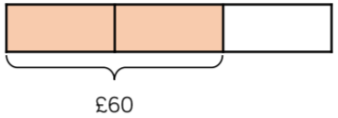
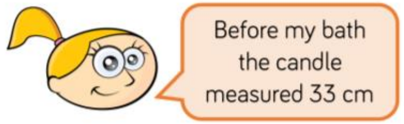
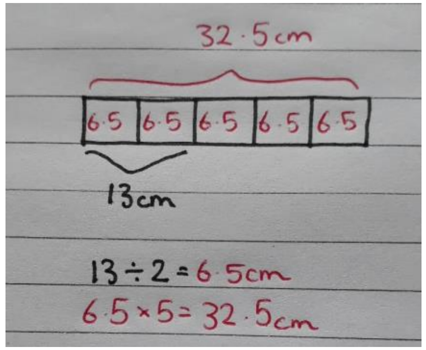
[Y5 white Rose document]

Fractions – fractions of amounts

<table>
<tr><th>Year 2</th><th>Year 3</th><th>Year 4</th><th>Year 5</th><th>Year 6</th></tr>
<tr>
<td>Concrete:

Pictorial:
</td>
<td>Concrete
Use place value counters instead of counting in ones when the 'whole' is large as it would be inefficient to use blank counters as 1s.

Pictorial:
Draw out the place value counters.
</td>
<td>Use the same concrete and pictorial methods as Y2 and Y3, depending on the numbers.

Progress to using knowledge of times tables to be able to use multiples as the parts.
</td>
<td>Become secure using the abstract method whilst representing this accurately as a bar model e.g.

Find $\frac{2}{7}$ of 42.

[Y5 White Rose document]</td>
<td>Confidently represent problems using bar models to show known and unknown information. Then use the abstract method to calculate the answer.
What is the value of A?
What is the value of B?

[Y6 White Rose document]
</td>
</tr>
</table>

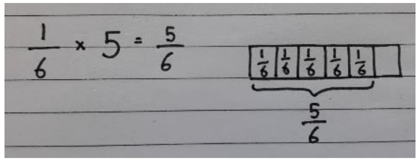
	• *Solve problems that include calculating the whole quantity.* This is $\frac{3}{4}$ of a set of beanbags.  How many were in the whole set? [Y3 White Rose document] There should be 4 rows, or parts, in total because the denominator is 4. Here, there are 3 equal rows, so another row of the same amount needs to be drawn.  	• *Solve problems that include calculating the whole quantity.* These three squares are $\frac{1}{4}$ of a whole shape.  [Y4 White Rose document] How many squares are in the whole shape? Children should identify that the whole has been divided into 4 parts and one part has 3 squares in it:  They know that each part is equal, so all the other parts also have 3 in them.  3 x 4 = 12 so there are 12 shapes in the whole. This could be introduced practically using squares of paper or cubes and drawing large bar models	• *Solve problems that include calculating the whole quantity* $\frac{7}{16}$ of a class are boys. There are 18 girls in the class. How many children are in the class? [Y5 White Rose document]  	• *Solve problems that include calculating the whole quantity.* Jack has spent $\frac{2}{3}$ of his money. He spent £60, how much did he have to start with?  Eva lit a candle while she had a bath. After her bath, $\frac{2}{5}$ of the candle was left. It measured 13 cm. Eva says:  Is she correct? Explain your reasoning. [Y6 White Rose document] She is incorrect because: 
To find percentages of amounts, the same bar modelling structures could be used as representations just substituting the percentages with fractions.				

Fractions – multiplying fractions

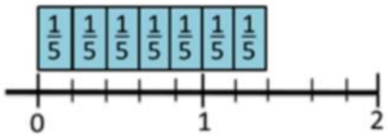
Year 5

- *multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams*

- *Multiply unit fractions by an integer*

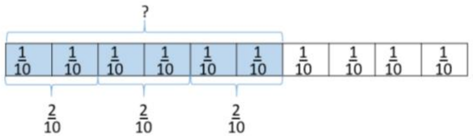


Similar method can be applied on a numberline, particularly if the fraction becomes greater than one.
$\frac{1}{5}$ x 7 =



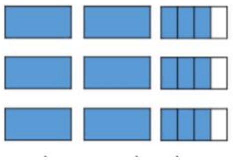
- *Multiply non-unit fractions by an integer*

Use the model to help you solve $3 \times \frac{2}{10}$



- *Multiply mixed numbers by integers*

Partition your fraction to help you solve $2\frac{3}{4} \times 3$



$2 \times 3 = 6$

$\frac{3}{4} \times 3 = \frac{9}{4} = 2\frac{1}{4}$

$6 + 2\frac{1}{4} = 8\frac{1}{4}$

[Y5 White Rose document]

Year 6

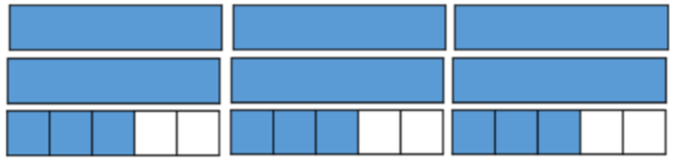
- *multiply simple pairs of proper fractions, writing the answer in its simplest form (e.g. 1 / 4 × 1 / 2 = 1 / 8)*
- *multiply fractions by integers (build on skills from Year 5)*

Eva partitions $2\frac{3}{5}$ to help her to calculate $2\frac{3}{5} \times 3$

$2 \times 3 = 6$

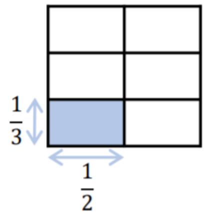
$\frac{3}{5} \times 3 = \frac{9}{5} = 1\frac{4}{5}$

$6 + 1\frac{4}{5} = 7\frac{4}{5}$

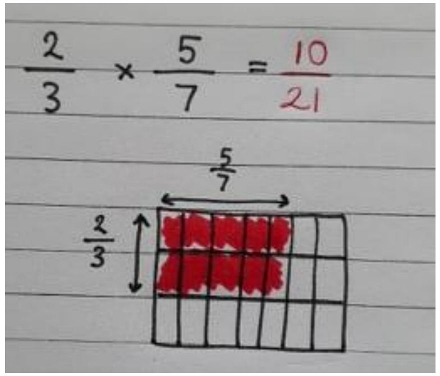


- *multiply fractions by fractions*

$\frac{1}{3} \times \frac{1}{2}$ is the same as $\frac{1}{3}$ of $\frac{1}{2}$



[Y6 White Rose document]



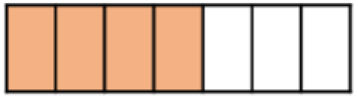
Fractions – dividing fractions

Year 6

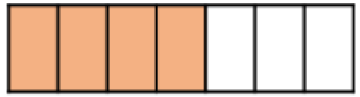
- *Divide fractions by integers*

  Dividing fractions where the numerator is a multiple of the integer they are dividing by.
  Use the sharing method of division.

$\frac{4}{7} \div 4 =$

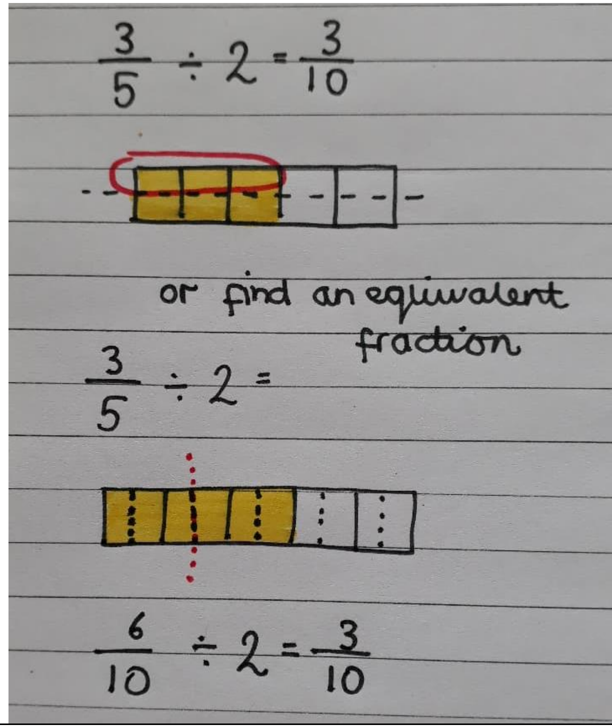


$\frac{4}{7} \div 2 =$

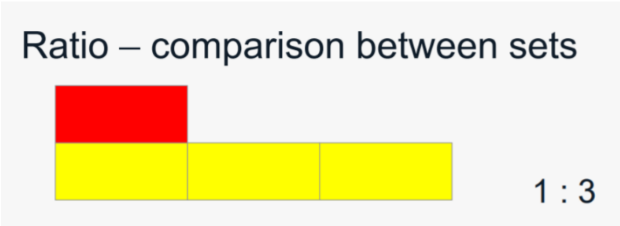


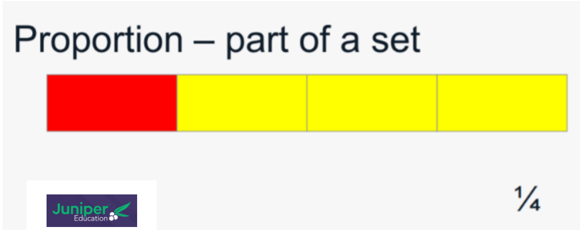
[Y6 White Rose document]

Dividing fractions where the numerator is NOT a multiple of the integer they are dividing by.
Use knowledge of equivalent fractions to create a fraction where the numerator IS a multiple of the integer they are dividing by.



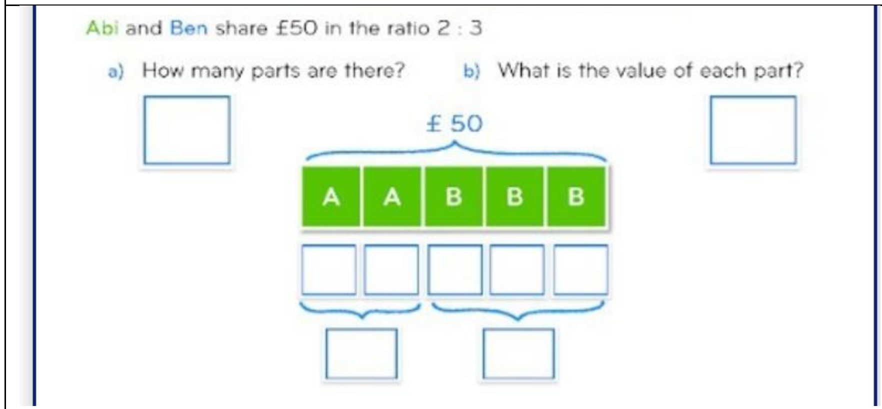
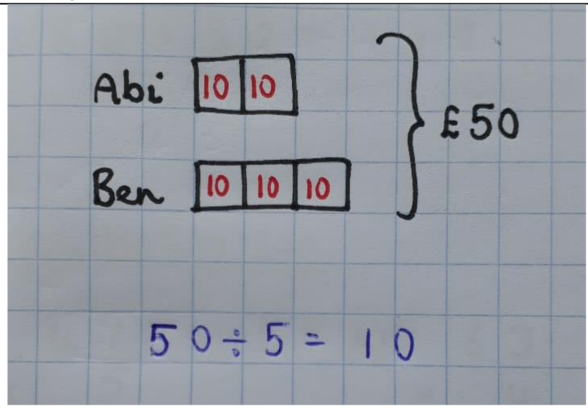
Ratio and proportion – Year 6





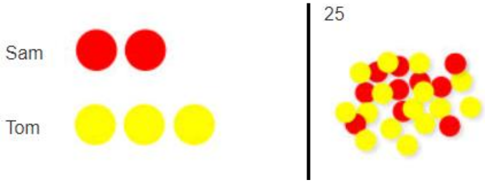
Structures of ratio and proportion are taught before Year 6 (but not explicitly as ratio) through the discussion of equal parts, sharing, and multiplication as repeated addition. The terminology of 'proportion' could be used before Year 6 when talking about fractions of wholes.

Division using ratio could be done using a 'one bar' method or 'comparative bars' (see below). We would encourage children to always use separative bars (the comparative method) because the different parts are easier to see and compare this way. Children who have particularly deep and secure understanding of ratio might be able to work flexibly and effectively using both.

One bar method:	**Comparative bars method:**
 [Third space learning]	

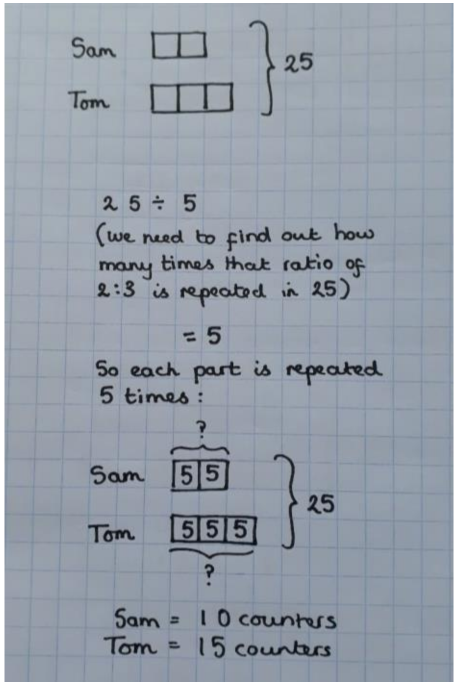
The following examples focus on using bar models to problem solve with ratio problems in Year 6. All of the following examples use ratios comparing two amounts but could easily be adapted for triple ratios e.g. 1:3:4. For examples of how to use bar models for proportion, visit 'fractions of amounts' as proportion means 'part of a whole'.

<table>
<tr>
<th>Finding the value of each part when the whole is known</th>
<th>Finding ‘how much more’ using ratio.</th>
<th>Finding a whole when a part is given:</th>
</tr>
<tr>
<td>
Sam and Tom have football stickers in the ratio of 2 to 3. Altogether they have 25 stickers. How many does Sam have? How many does Tom have?

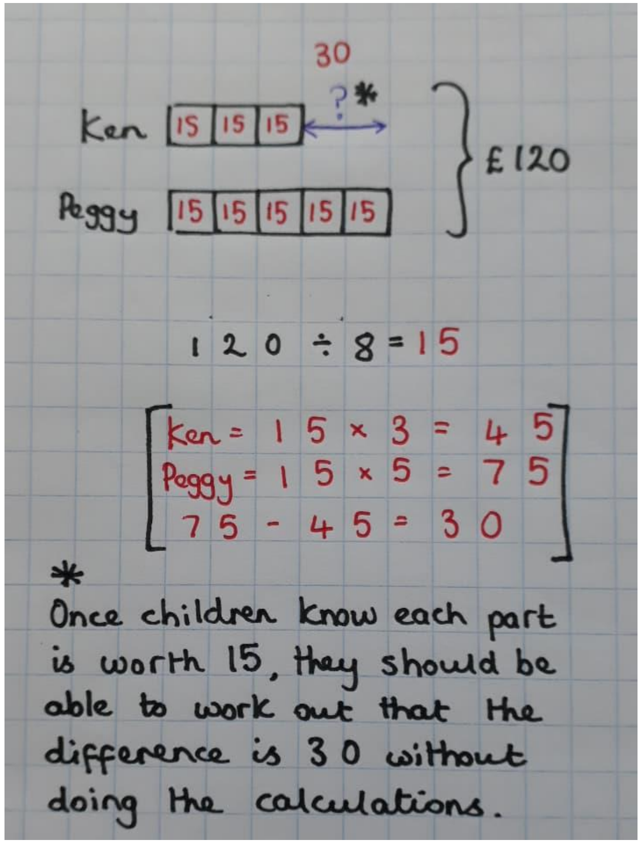


National Centre for Excellence in the Teaching of Mathematics

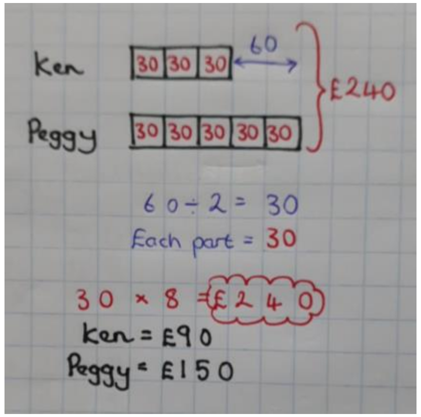
As a bar model:



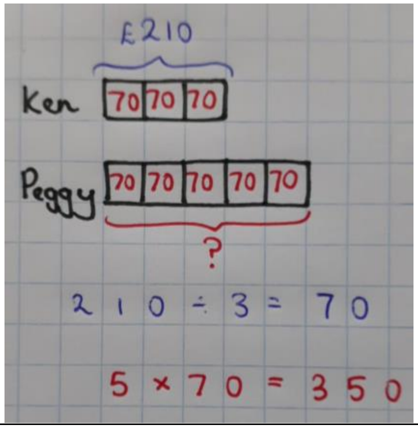
</td>
<td>
Ken and Peggy share £120 in the ratio 3:5. How much more does Peggy have than Ken?



</td>
<td>
Ken and Peggy share some money in the ratio 3:5. Peggy gets £60 more than Peggy. How much did they share? How much did they get each?



Ken and Peggy share some money in the ratio 3:5. Ken has £210 pounds. How much does Peggy have?



</td>
</tr>
</table>

Algebra

Defined as: knowing and applying the rules of calculation to find unknown variables and patterns.

Years 1 & 2

Use bar models to solve missing number questions

e.g.

___ + 5 = 13.

Discuss the known and unknown

13	
?	5

This will help children develop algebraic thinking regarding 'the unknown value' and also build their understanding of using inverse relationships to support their algebraic problem solving.

Years 2 & 3

Use bar models to explore the equals sign as a balance point rather than 'on the right'

e.g.

54 = 25 + ___

Discuss the known and unknown parts.

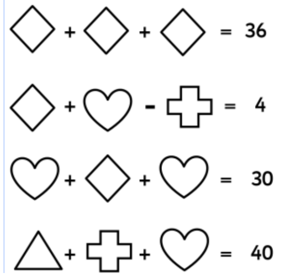
54	
?	25

Year 4 & 5

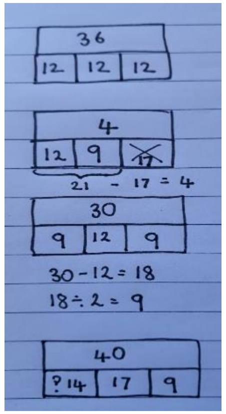
Use bar models to help solve picture problems using the four operations.

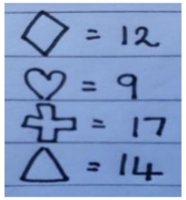
e.g.

Work out the value of each shape



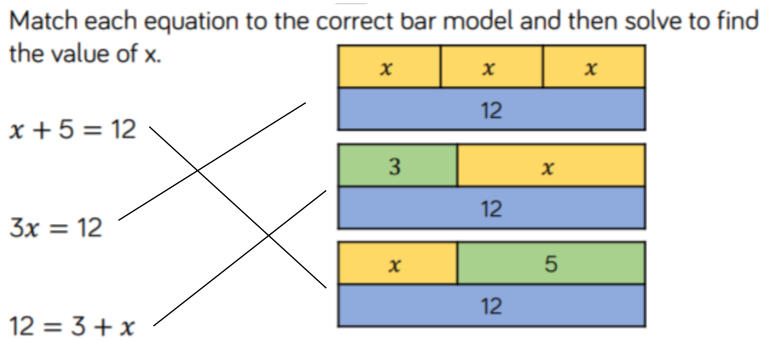
[Classroom secrets example]



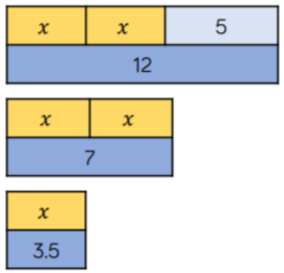


Year 6

Represent algebraic expressions using bar models and use the structure to help work out the answers.



[Y6 White Rose document]



$2x + 5 = 12$

Remove 5 from both sides of the equation (balance the sides).

$2x = 7$

Half x

$x = 3.5$

2

[Y6 White Rose document]

Children should not become reliant on using bar models to solve equations. The most efficient method is to solve them algebraically.

Measurement

Measurement encompasses: time, money, weight/mass, length/height, capacity/volume, area and perimeter, conversions.

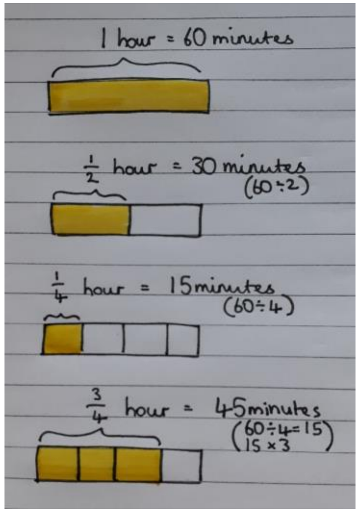
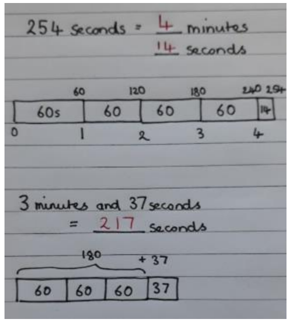
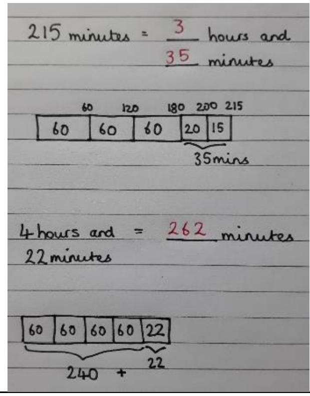
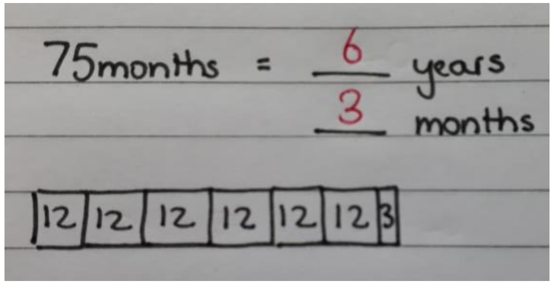
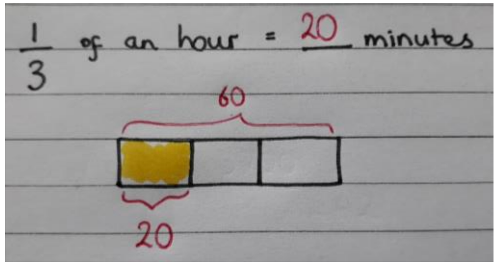
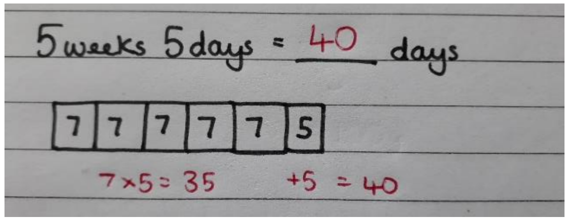
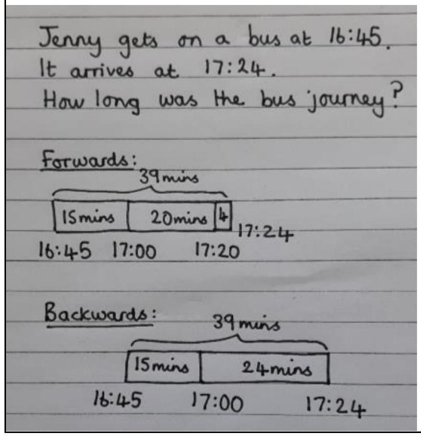
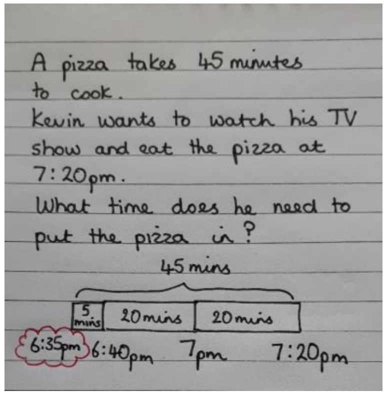
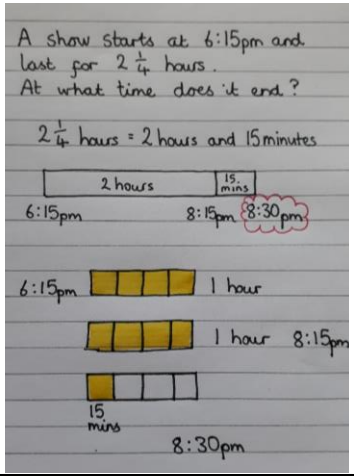
For most areas of measurement, all of the above bar modelling structures explored in this document can be manipulated and applied to calculations and problems where the values are units of measure.

For example:

- If adding values of money, refer to the year group's appropriate addition bar model structures.
- If finding a fraction of a length, refer to the year group's appropriate fraction of amounts bar model structures.
- If multiplying the mass of an object, refer to the year group's appropriate multiplication bar model structures.

Further structures can be used when problem solving with time and also when converting between units of measure (see below).

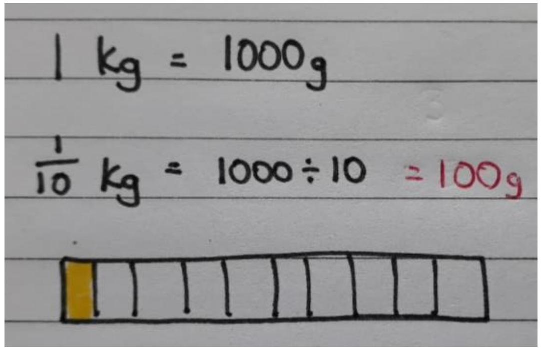
Measurement: Time

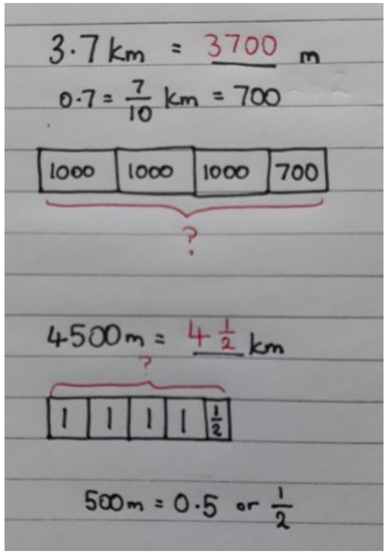
<table>
<tr><th>Year 3</th><th>Year 4</th><th colspan="2">Year 5 / Year 6</th></tr>
<tr>
<td>Use bar models to help understand half and quarter hours and their relationship to being a fraction of 60 minutes.
</td>
<td>Use bar models to help represent conversion between:
- Hours and minutes, based on the understand that 1 hour is 60 minutes
- minutes and seconds, based on the understanding 1 minute being 60 seconds.

</td>
<td rowspan="2" colspan="2">Use bar modelling skills learned in Years 3 and 4 and apply these to reading time tables where durations need to be found or measures need to be converted between hours/minutes/seconds.


</td>
</tr>
<tr>
<td colspan="2">Solve duration problems including calculating the duration or finding start and end times.


</td>
</tr>
</table>

Measurement: converting units

Year 5

Children will need to be secure in their conversion between fractions and decimals to use these representations accurately.

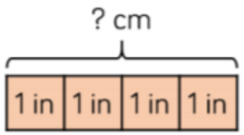


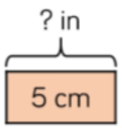


These representations can be applied to mass, length and capacity (including millimetres/milligrams) as a precursor for the abstract method of calculating the conversions.

- *Converting imperial units*

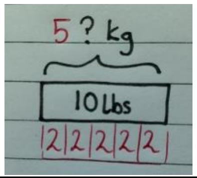
One inch is approximately 2.5 centimetres
1 inch ≈ 2.5 cm

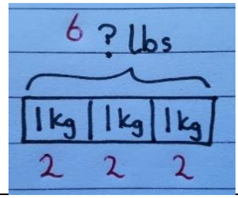




1 kilogram is approximately 2 pounds
1 kg ≈ 2 lbs

[Y5 White Rose document]

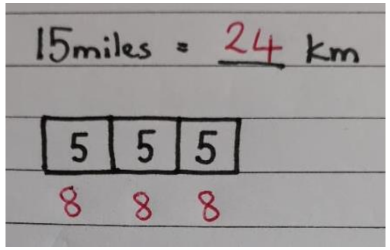




Year 6

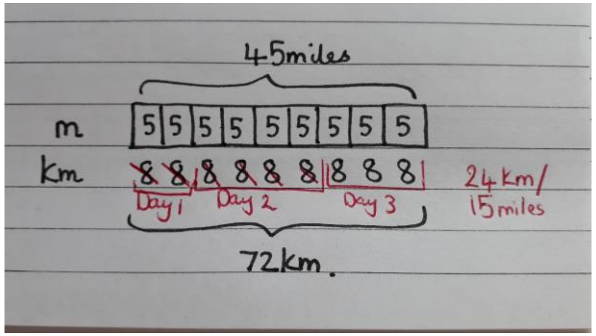
5 miles ≈ 8 kilometres

[Y6 White Rose document]



Applying conversions to problems.

Mo cycles 45 miles over the course of 3 days. On day 1, he cycles 16 km. On day 2, he cycles 10 miles further than he did on day 1 How far does he cycle on day 3? Give your answer in miles and in kilometres.
[Y6 White Rose document]



References

Thank you to the following sources of information that enabled the compilation of this document.

THIRD SPACE LEARNING

- https://thirdspacelearning.com/blog/teach-bar-model-method-arithmetic-maths-word-problems-ks1-ks2/
- The Ultimate Guide to Bar Modelling https://thirdspacelearning.com/resources/resource-ultimate-guide-bar-modelling/

Juniper Education

Primary Mathematics: Effective teaching of Ratio and Proportion. Online course [Paul Hargreaves]

National Centre for Excellence in the Teaching of Mathematics

https://www.ncetm.org.uk/Default.aspx?page=13&module=res&mode=100&resid=44565&

White Rose Maths

- https://whiterosemaths.com/resources/classroom-resources/barvember/
- https://whiterosemaths.com/resources/schemes-of-learning/primary-sols/

Burlish Park Primary School

http://www.burlishpark.co.uk/wp-content/uploads/2018/11/bar-model-progression.pdf

MathsHUBS White Rose 'The importance of bar modelling' session slides.

Classroom secrets

https://classroomsecrets.co.uk/year-6-algebra-worksheet-shape-puzzles/

For further information or questions, contact the document author at: lrwilson14@yahoo.co.uk